Cubic NK-SVD: An Algorithm for Designing Parametric Dictionary in Frequency Estimation

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Presentation Overview

1. Introduction
2. Cubic NK-SVD Algorithm
3. Convergence
4. Simulations
5. Concluding Remarks

Outline

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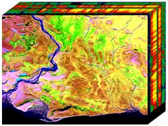
Applications

(a)



(b)

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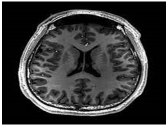
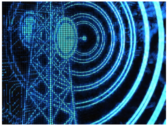


(d)



(e)

(f)

- Source localization in MRI.
- DOA estimation in wireless communications.
- Frequency and amplitude estimation in spectrum analysis.
- Doppler estimation in radar systems.

Line Spectral Estimation (LSE)

- Consider the LSE problem

$$\boldsymbol{y}(t) = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s}(t) + \boldsymbol{e}(t), \quad t = 1, \cdots, T, \tag{1}$$

where $\boldsymbol{y}(t) = [y_1(t), \cdots, y_N(t)]^T$, $\boldsymbol{\theta} = [\theta_1, \cdots, \theta_K]^T$, $\boldsymbol{s}(t) = [s_1(t), \cdots, s_K(t)]^T$, $\boldsymbol{e}(t) = [e_1(t), \cdots, e_N(t)]^T$.

- The matrix $\boldsymbol{A}(\boldsymbol{\theta}) = [\boldsymbol{a}(\theta_1), \cdots, \boldsymbol{a}(\theta_K)]^T$ is an array manifold matrix and $\boldsymbol{a}(\theta_k)$ is called steering vector of the k-th source.

$$\boldsymbol{a}(\theta) = \frac{1}{\sqrt{N}} \left[1, e^{j\theta} \cdots, e^{j(N-1)\theta}\right]^T, \tag{2}$$

Line Spectral Estimation (LSE)

- The model (1) can be rewritten in a matrix form as

$$\boldsymbol{Y} = \boldsymbol{A}(\theta)\boldsymbol{S} + \boldsymbol{E}, \tag{3}$$

  where $\boldsymbol{Y} = [\boldsymbol{y}(1), \cdots, \boldsymbol{y}(T)]$ is the observation matrix consisting of T observed vectors, $\boldsymbol{S} = [\boldsymbol{s}(1), \cdots, \boldsymbol{s}(T)]$, and $\boldsymbol{E}$ denotes the noise matrix.[1]
- **The goal** is to estimate the number of sources K, the unknown frequencies θ, and gains $\boldsymbol{S}$ given the observed data $\boldsymbol{Y}$ and the sensing matrix $\boldsymbol{\Phi}$.

[1] The issue also arises in the context of CS reconstruction.

Existing Works

- Subspace methods: MUSIC[2], ESPRIT[3].
  - Complete measurement;
  - A sufficient number of snapshots;
  - The model order is known;
  - Hign signal-to-noise ratio (SNR).
- On-grid methods: ℓ_1-SVD[4]
  - Grid mismatch[5].

[2] R. Schmidt. "Multiple Emitter Location and Signal Parameter Estimation". In: *IEEE Trans. Antennas Propag.* 34.3 (1986), pp. 276–280.

[3] R. Roy and T. Kailath. "ESPRIT–Estimation of Signal Parameters via Rotational Invariance Techniques". In: *IEEE Trans. Acoust., Speech, Signal Process.* 37.7 (1989), pp. 984–995.

[4] D. Malioutov, M. Cetin, and A. S. Willsky. "A Sparse Signal Reconstruction Perspective for Source Localization with Sensor Arrays". In: *IEEE Trans. Signal Process.* 53.8 (2005), pp. 3010–3022.

[5] Y. Chi et al. "Sensitivity to Basis Mismatch in Compressed Sensing". In: *IEEE Trans. Signal Process.* 59.5 (2011), pp. 2182–2195.

Existing Works

To combat the severity of the grid mismatch

- Off-grid methods: Sure-IR[6] NOMP[7], Bayesian methods[8][9][10].
    - Lack convergence analysis.
- Gridless methods: ANM[11], EMaC[12].
    - Involving semidefinite programming (SDP), which can result in significant computational complexity.

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[6] J. Fang et al. "Super-Resolution Compressed Sensing for Line Spectral Estimation: An Iterative Reweighted Approach". In: *IEEE Trans. Signal Process.* 64.18 (2016), pp. 4649–4662.

[7] B. Mamandipoor, D. Ramasamy, and U. Madhow. "Newtonized Orthogonal Matching Pursuit: Frequency Estimation over the Continuum". In: *IEEE Trans. Signal Process.* 64.19 (2016), pp. 5066–5081.

[8] Z. Yang, L. Xie, and C. Zhang. "Off-Grid Direction of Arrival Estimation Using Sparse Bayesian Inference". In: *IEEE Trans. Signal Process.* 61.1 (2012), pp. 38–43.

[9] T. L. Hansen et al. "A Sparse Bayesian Learning Algorithm with Dictionary Parameter Estimation". In: *Proc. IEEE Sensor Array Multichannel Signal Process. Workshop (SAM)*. 2014, pp. 385–388.

[10] T. L. Hansen, B. H. Fleury, and B. D. Rao. "Superfast Line Spectral Estimation". In: *IEEE Trans. Signal Process.* 66.10 (2018), pp. 2511–2526.

[11] G. Tang et al. "Compressed Sensing off the Grid". In: *IEEE Trans. Inf. Theory* 59.11 (2013), pp. 7465–7490.

[12] Y. Chen and Y. Chi. "Spectral Compressed Sensing via Structured Matrix Completion". In: *Proc. Int. Conf. Mach. Learn. (ICML)*. 2013, pp. 414–422.

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Parametric Dictionary Learning

- The LSE problem can be formulated as a parametric DL problem

$$\begin{aligned} &\min_{\tilde{\theta},\boldsymbol{X}} \|\boldsymbol{Y} - \boldsymbol{D}(\tilde{\theta})\boldsymbol{X}\|_F^2 \\ &\text{s.t.} \quad \|\boldsymbol{x}_i\|_0 \leq K,\ \forall i = 1, \cdots, T, \end{aligned} \tag{4}$$

  where $\boldsymbol{x}_i$ is the i-th column of the matrix $\boldsymbol{X}$.
- Inspired by classical K-SVD algorithm [12], we propose a parametric DL algorithm for LSE by incorporating cubic regularization into Newton refinements.

Algorithmic Framework

- Sparse coding stage:
  - OMP[13], FISTA[14].
- Atom update stage:
  - To minimize the following objective

$$
\begin{aligned}
S(\tilde{\theta}_k, \boldsymbol{x}_T^k) &= \| \boldsymbol{Y} - \boldsymbol{D}(\tilde{\theta})\boldsymbol{X} \|_F^2 \\
&= \| \boldsymbol{Y} - \sum_{j=1}^{R} \boldsymbol{a}(\tilde{\theta}_j)\boldsymbol{x}_T^j \|_F^2 \\
&= \| (\boldsymbol{Y} - \sum_{j \neq k} \boldsymbol{a}(\tilde{\theta}_j)\boldsymbol{x}_T^j) - \boldsymbol{a}(\tilde{\theta}_k)\boldsymbol{x}_T^k \|_F^2 \\
&= \| \boldsymbol{E_k} - \boldsymbol{a}(\tilde{\theta}_k)\boldsymbol{x}_T^k \|_F^2,
\end{aligned} \tag{5}
$$

[13] J. A. Tropp and A. C. Gilbert. “Signal Recovery from Random Measurements via Orthogonal Matching Pursuit”. In: *IEEE Trans. Inf. Theory* 53.12 (2007), pp. 4655–4666.

[14] A. Beck and M. Teboulle. “A Fast Iterative Shrinkage-Thresholding Algorithm for Linear Inverse Problems”. In: *SIAM J. Imaging Sci.* 2.1 (2009), pp. 183–202.

Atom Update

- Directly optimizing $S(\tilde{\theta}_k, \boldsymbol{x}_T^k)$ over $\tilde{\theta}_k$ and $\boldsymbol{x}_T^k$ is difficult.
  1. Coarse estimate stage.
  2. Cubic Newton refinement stage.
- Coarse estimate stage:
  - Use SVD to find the closest rank-1 matrix (in Frobenius norm) that approximates $\boldsymbol{E_k}$.
  - Restrict it to a finite discrete set denoted by $\boldsymbol{\Omega} = \{k(2\pi/\gamma N) : k = 0, 1, \cdots, (\gamma N - 1)\}$.
  - The output $(\tilde{\theta}_k)_c$ of this stage is the solution to the following optimization problem

$$(\tilde{\theta}_k)_c = \arg\min_{\theta \in \boldsymbol{\Omega}} |\boldsymbol{a}(\theta)^H \boldsymbol{u}_1|. \tag{6}$$

  - Given $\tilde{\theta}_k$ is fixed, the optimal coefficients that minimizes $S(\tilde{\theta}_k, \boldsymbol{x}_T^k)$ is given by

$$\hat{\boldsymbol{x}}_R^k = \boldsymbol{a}(\tilde{\theta}_k)^H \boldsymbol{E}_{\boldsymbol{k}}^{\boldsymbol{R}}, \tag{7}$$

Atom Update

- Cubic Newton refinement stage:
  - Note that the partial Hessian of function S with respect to θ is Lipschitz continuous.
  - We can refine the frequency using cubic regularization of Newton's method[15]:

$$\hat{\theta}_k' \in \arg\min_{\theta} \xi_{\hat{\theta}_k, \hat{\boldsymbol{x}}_T^k}^{L(\hat{\boldsymbol{x}}_T^k)}(\theta), \tag{8}$$

where the auxiliary function

$$\begin{aligned} \xi_{\hat{\theta}_k, \hat{\boldsymbol{x}}_T^k}^{L(\hat{\boldsymbol{x}}_T^k)}(\theta) = & \left\langle \nabla_\theta S\left(\hat{\theta}_k, \hat{\boldsymbol{x}}_T^k\right), \theta - \theta_k \right\rangle \\ & + \frac{1}{2} \left\langle \nabla_{\theta\theta}^2 S\left(\hat{\theta}_k, \hat{\boldsymbol{x}}_T^k\right)(\theta - \theta_k), \theta - \theta_k \right\rangle \\ & + \frac{L(\hat{\boldsymbol{x}}_T^k)}{6} |\theta - \theta_k|^3. \end{aligned} \tag{9}$$

[15] Y. Nesterov and B. T. Polyak. "Cubic Regularization of Newton Method and Its Global Performance". In: *Math. Program.* 108.1 (2006), pp. 177–205.

Proposed Algorithm

Algorithm Cubic Newtonized K-SVD (Cubic NK-SVD)

Input: $\boldsymbol{Y}$, R, γ, ϵ.
Initialize: $\widehat{K} = R$, $\boldsymbol{f} = \{k(2\pi/R) : k = 0, 1, \cdots, R-1\}$ and $\boldsymbol{\Omega} = \{k(2\pi/\gamma N) : k = 0, 1, \cdots, (\gamma N - 1)\}$.
Output: Updated $\widehat{K}$, $\boldsymbol{f}$, and $\boldsymbol{X}$.
repeat
    Sparse Coding Stage:
    $\boldsymbol{X}$ is updated using OMP with an error bound ϵ.
    Atom Update Stage:
    for $k = 1$ **to** R **do**
        Compute $\mathcal{I}_k = \{\ell \mid \boldsymbol{x}_T^k(\ell) \neq 0\}$.
        if $\mathcal{I}_k = \emptyset$ **then**
            Eliminate the unused atom $\boldsymbol{a}(\tilde{\theta}_j)$.
            $\widehat{K} = \widehat{K} - 1$.
        else
            Compute error $\boldsymbol{E_k} = \boldsymbol{Y} - \sum_{j \neq k} \boldsymbol{a}(\tilde{\theta}_j)\boldsymbol{x}_T^j$.
            Retrict $\boldsymbol{E_k}$ by $\boldsymbol{E_k^R} = (\boldsymbol{E_k})_{\mathcal{I}_k}$.
            Compute first left singular vector $\boldsymbol{u}_1$ of $\boldsymbol{E_k^R}$.
            Coarse Estimate: Update $(\tilde{\theta}_k)_c$ within $\boldsymbol{\Omega}$ by (6) and its corresponding gain $(\boldsymbol{x}_T^k)_c$ by (7).
            Cubic Newton Refinement: Refine $(\hat{\theta}_k, \hat{\boldsymbol{x}}_T^k)$ using (8).
        end if
    end for
until stopping criterion met

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Convergence

- Focus on the nonconvex minimization problem involving two blocks of variables, $(x, \boldsymbol{y}) \in \mathbb{R}^1 \times \mathbb{C}^T$:

$$\min_{x,\boldsymbol{y}} H(x, \boldsymbol{y}). \tag{10}$$

Assumption

(i) Assume that the objective function $H(x, \boldsymbol{y})$ is bounded below, i.e., $\inf_{\mathbb{R}^1 \times \mathbb{C}^T} H \geq H^*$.
(ii) For any fixed $\boldsymbol{y}$ the function $x \to H(x, \boldsymbol{y})$ is $C^{2,2}_{L(\boldsymbol{y})}$, namely the partial Hessian $\nabla^2_{xx} H(x, \boldsymbol{y})$ is Lipschitz continuous with moduli $L(\boldsymbol{y})$, that is

$$\left\| \nabla^2_{xx} H(x_1, \boldsymbol{y}) - \nabla^2_{xx} H(x_2, \boldsymbol{y}) \right\| \leq L(\boldsymbol{y}) \|x_1 - x_2\|, \quad \forall x_1, x_2 \in \mathbb{R}^1.$$

Convergence

Assumption

(iii) For any fixed x the function $\boldsymbol{y} \to H(x, \boldsymbol{y})$ is assumed to be strongly convex (or, more specifically, ν-strongly convex) in $\mathbb{C}^T$. In other words, there exists $\nu > 0$ such that

$$(\nabla^2_{\boldsymbol{yy}})_c H(x, \boldsymbol{y}) \succeq \frac{\nu}{2} \boldsymbol{I}_{2T} \succ \boldsymbol{0},$$

where we used the notation $(\nabla^2_{\boldsymbol{yy}})_c H(\boldsymbol{y})$ to refer to the complex form of the partial Hessian matrix of $H(x, \boldsymbol{y})$.
(iv) There exists λ^-, λ^+ such that

$$\inf \left\{ L\left(\boldsymbol{y}^k\right) : k \in \mathbb{N} \right\} \geq \lambda^-, \tag{11}$$

$$\sup \left\{ L\left(\boldsymbol{y}^k\right) : k \in \mathbb{N} \right\} \leq \lambda^+. \tag{12}$$

Convergence

- As outlined in Algorithm 1, we generate a sequence $\{(x^k, \boldsymbol{y}^k)\}_{k\in\mathbb{N}}$ via the scheme

$$x^{k+1} \in \arg\min_{x} \xi_{x^k,\boldsymbol{y}^k}^{L(\boldsymbol{y}^k)}(x), \tag{13}$$

$$\boldsymbol{y}^{k+1} \in \arg\min_{\boldsymbol{y}} H\left(x^{k+1}, \boldsymbol{y}\right), \tag{14}$$

where the auxiliary function

$$\begin{aligned} \xi_{x^k,\boldsymbol{y}^k}^{L(\boldsymbol{y}^k)}(x) = &\left\langle \nabla_x H\left(x^k, \boldsymbol{y}^k\right), x - x^k \right\rangle \\ &+ \frac{1}{2}\left\langle \nabla_{xx}^2 H\left(x^k, \boldsymbol{y}^k\right)(x - x^k), x - x^k \right\rangle \\ &+ \frac{L(\boldsymbol{y}^k)}{6}|x - x^k|^3. \end{aligned} \tag{15}$$

Basic Properties

Lemma ([15])

Let $h : \mathbb{R}^1 \to \mathbb{R}$ be a twice differentiable function with Hessian assumed L_h-Lipschitz continuous. Then,

$$\left|h'(x_2) - h'(x_1) - h''(x_1)(x_2 - x_1)\right| \leq \frac{1}{2} L_h |x_2 - x_1|^2, \tag{16}$$

$$\begin{aligned} &\Big| h(x_2) - h(x_1) - \left\langle h'(x_1), x_2 - x_1 \right\rangle \\ &\quad - \frac{1}{2} \left\langle h''(x_1)(x_2 - x_1), x_2 - x_1 \right\rangle \Big| \leq \frac{L_h}{6} |x_2 - x_1|^3. \end{aligned} \tag{17}$$

Basic Properties

Lemma ([15])

For any $x \in \mathbb{R}^1$ *we have*

$$\bar{h}_{L_h}(x) \leq \min_{\tilde{x}} \left[h(\tilde{x}) + \frac{L_h}{3} |\tilde{x} - x|^3 \right], \tag{18}$$

$$h(x) - \bar{h}_{L_h}(x) \geq \frac{L_h}{12} |T_{L_h}(x) - x|^3. \tag{19}$$

Moreover, we have

$$h(T_{L_h}(x)) \leq \bar{h}_{L_h}(x). \tag{20}$$

Lemma ([15])

$T_{L_h}(x)$ satisfies

$$|h'(T_{L_h}(x))| \leq L_h |T_{L_h}(x) - x|^2. \tag{21}$$

Lemma ([16])

Let $f : \mathbb{C}^T \to \mathbb{R}$ be a continuously differentiable function that is ν-strongly convex on $\mathbb{C}^T$ with $\nabla f(\boldsymbol{y}^*) = 0$. Then,

$$f(\boldsymbol{y}) \geq f(\boldsymbol{y}^*) + \frac{\nu}{2}\|\boldsymbol{y} - \boldsymbol{y}^*\|^2, \quad \forall \boldsymbol{y} \in \mathbb{C}^T. \tag{22}$$

Lemma

Suppose that Assumption 1 hold. Let $\{\boldsymbol{z}^k\}_{k\in\mathbb{N}}$ *be a sequence iteratively generated by* (13) *and* (14). *The following assertions hold.*
(i) The sequence $\{H(\boldsymbol{z}^k)\}_{k\in\mathbb{N}}$ *is nonincreasing, and in particular, for any* $k \geq 0$*, we have*

$$\frac{\lambda^-}{12}\left|x^{k+1}-x^k\right|^3+\left\|\boldsymbol{y}^{k+1}-\boldsymbol{y}^k\right\|^2 \leq H\left(\boldsymbol{z}^k\right)-H\left(\boldsymbol{z}^{k+1}\right). \tag{23}$$

(ii) We have

$$\sum_{k=0}^{\infty}\left|x^{k+1}-x^k\right|^3+\left\|\boldsymbol{y}^{k+1}-\boldsymbol{y}^k\right\|_3^3=\sum_{k=0}^{\infty}\left\|\boldsymbol{z}^{k+1}-\boldsymbol{z}^k\right\|_3^3<\infty, \tag{24}$$

and hence $\lim_{k\to\infty}\left\|\boldsymbol{z}^{k+1}-\boldsymbol{z}^k\right\|=0$.

A Gradient Lower Bound

Lemma

Suppose that Assumptions 1 hold. Let $\{\boldsymbol{z}^k\}_{k\in\mathbb{N}}$ *be a sequence generated by* (13) *and* (14) *which is assumed to be bounded, i.e., there exists* σ *such that* $\|\boldsymbol{z}^k\| \leq \sigma, \forall k \in \mathbb{N}$. *For each positive integer k, we have*

$$\|\nabla_x H(x^{k+1}, \boldsymbol{y}^{k+1})\| \leq \lambda^+ \|x^{k+1} - x^k\|^2 + 2\sigma \|\boldsymbol{y}^{k+1} - \boldsymbol{y}^k\|, \tag{25}$$

and Wirtinger derivative

$$\|\nabla_{\boldsymbol{y}} H(x^{k+1}, \boldsymbol{y}^{k+1})\| = \mathbf{0}. \tag{26}$$

Then,

$$\begin{aligned} \|\nabla_{\boldsymbol{z}} H(\boldsymbol{z}^{k+1})\| &\leq \|\nabla_x H(x^{k+1}, \boldsymbol{y}^{k+1})\| + \|\nabla_{\boldsymbol{y}} H(x^{k+1}, \boldsymbol{y}^{k+1})\| \\ &\leq \rho_2 \|\boldsymbol{z}^{k+1} - \boldsymbol{z}^k\|, \end{aligned} \tag{27}$$

where $\rho_2 = \max\{\lambda^+, 2\sigma\}$.

Properties of The Limit Point Set

- The set of all limit points is denoted by $\omega(\boldsymbol{z}^0)$, i.e.,

$$\omega(\boldsymbol{z}^0) = \{\bar{\boldsymbol{z}} \in \mathbb{R}^1 \times \mathbb{C}^T : \exists \text{ an increasing sequence of integers } \{k_l\}_{l\in\mathbb{N}}, \text{ sucn that } \boldsymbol{z}^{k_l} \to \bar{\boldsymbol{z}} \textit{ as } l \to \infty\}.$$

Theorem

Suppose that Assumption 1 hold. Let $\{\boldsymbol{z}^k\}_{k\in\mathbb{N}}$ be a sequence generated from the cubic NK-SVD algorithm which is assumed to be bounded. The following assertions hold.
(i) $\emptyset \neq \omega(\boldsymbol{z}^0) \in crit\, H := \{\boldsymbol{z} : \nabla_{\boldsymbol{z}} H(\boldsymbol{z}) = 0\}$.
(ii) We have

$$\lim_{k\to\infty} dist(\boldsymbol{z}^k, \omega(\boldsymbol{z}^0)) = 0, \tag{28}$$

where $dist(\boldsymbol{z}^k, \omega(\boldsymbol{z}^0))$ denotes the distance from $\boldsymbol{z}^k$ to $\omega(\boldsymbol{z}^0)$.

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Experimental Setup

- We consider a mixture of K sinusoids of length $N = 64$.
- The true frequencies $\{\theta_k, k = 1, \cdots, K\}$ are uniformly generated over $[0, 2\pi)$.
- The true coefficients in $\boldsymbol{s}(t)$ are generated i.i.d. with uniform random phase on $[0, 2\pi)$ and amplitudes drawn from a normal density of mean 10 and variance 3.
- Evaluation Measures:

$$\mathrm{RSNR} = 20 \log_{10} \left(\frac{\|\boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s}(t)\|_2}{\left\|\boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s}(t) - \boldsymbol{A}(\hat{\boldsymbol{\theta}})\hat{\boldsymbol{s}}(t)\right\|_2} \right), \tag{29}$$

$$\beta(\boldsymbol{\theta}, \hat{\boldsymbol{\theta}}) = \frac{1}{K} \sum_{k=1}^{K} (\min_{\hat{\theta} \in \hat{\boldsymbol{\theta}}} d(\hat{\theta}, \theta_k))^2. \tag{30}$$

Frequency Estimation for SMV Model

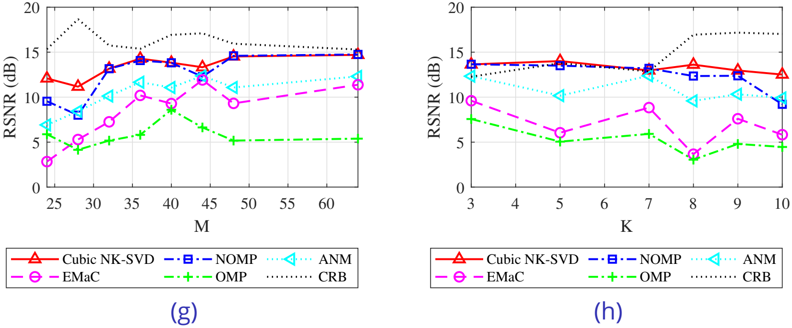
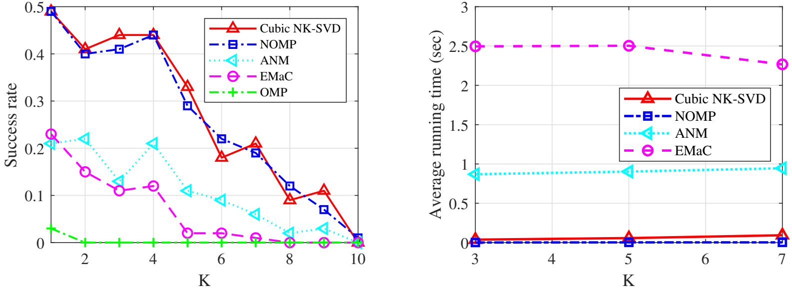


Figure: RSNRs of respective algorithms. (a) RSNRs vs. M, $K = 7$ and PSNR=10 dB. (b) RSNRs vs. K, $M = 32$ and PSNR=10 dB.

Frequency Estimation for SMV Model



(a) (b)

Figure: (a) Success rates of respective algorithms vs. K, $M = 32$ and PSNR=10 dB. (b) Average running times (sec) of respective algorithms when $M = 24$ and PSNR=10 dB.

Frequency Recovery in Low SNR Scenerios

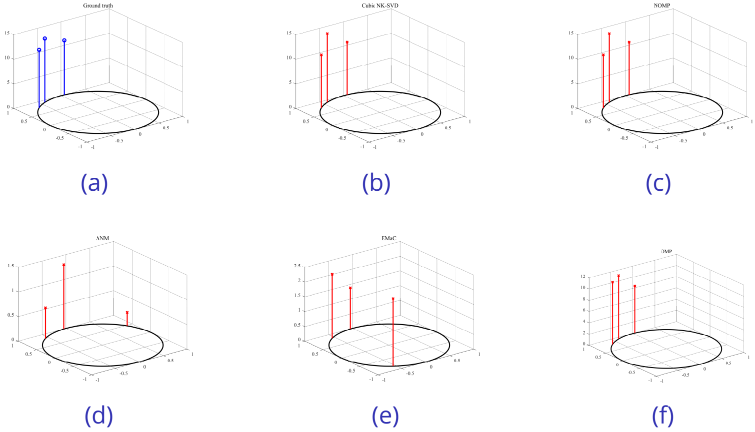


Figure: Frequency estimation using different algorithms when $M = 24$, $K = 3$ and PSNR=0 dB. (a) Ground truth. (b) Cubic NK-SVD. (c) NOMP. (d) ANM. (e) EMaC. (f) OMP.

Ability to Recover Closely-Spaced Frequencies

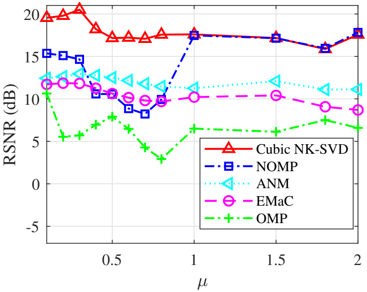
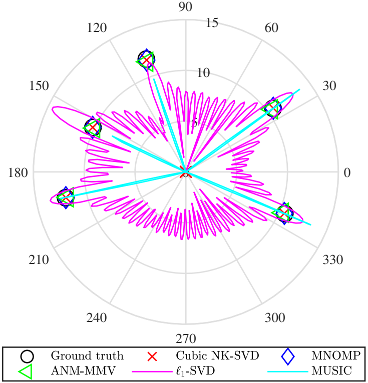
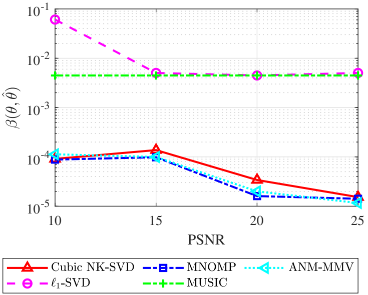


Figure: RSNRs of respective algorithms vs. μ, $M = 32$, $K = 2$ and PSNR=10 dB.

Frequency Estimation for MMV Model



(a)



(b)

Figure: (a) Angular spectra obtained using different algorithms when $M = 64$, $K = 5$ and PSNR=20 dB. (b) $\beta(\boldsymbol{\theta}, \hat{\boldsymbol{\theta}})$ of respective algorithms vs. PSNR, $M = 64$, $T = 48$, and $K = 5$.

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Concluding Remarks

- We propose a novel parametric DL algorithm for LSE, applicable in both SMV and MMV scenarios.
- We rigorously establish the convergence of the proposed algorithm within the BCD framework.
- Extensive simulations demonstrate that cubic NK- SVD outperforms existing SOTA methods in both SMV and MMV settings.

Thanks for Your Attention!

Email: xzliu@buaa.edu.cn

https://arxiv.org/abs/2408.03708

https://github.com/xzliu-opt/Cubic-NK-SVD